$\frac{1}{j} = \frac{1}{\sqrt{-1}} \cdot \frac{\sqrt{-1}}{\sqrt{-1}} = \frac{\sqrt{-1}}{-1} = -j$

Polar Form

$c = r \cdot e^{j\theta} = r\measuredangle\theta$

Conversion Between Rectangular and Polar

$r = \sqrt{x^2 + y^2}$ $\qquad y = rsin(\theta)$

$\theta = tan^{-1}(\frac{y}{x})$ $\qquad x = rcos(\theta)$

Euler's Formula:

$e^{j\theta} = cos(\theta) + j \cdot sin(\theta)$

Useful Identities:

$e^{j90^\circ} = e^{j\frac{\pi}{2}} = j$

$e^{j\pi} = -1$

For $c = x + jy = re^{j\theta}$

$c_* = complex\ conjugate\ of\ c = x - jy = re^{-j\theta}$

Laplace transforms

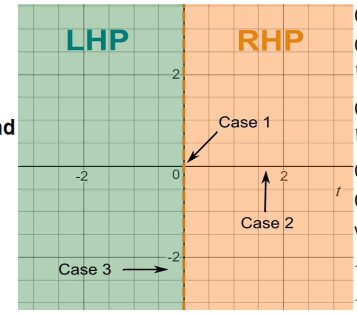
$\mathcal{L}\{\delta(t)\} = 1$ $\qquad \mathcal{L}\{te^{\lambda t}u(t)\} = \frac{1}{(s-\lambda)^2}$

$\mathcal{L}\{u(t)\} = \frac{1}{s}$ $\qquad \mathcal{L}\{t^n e^{\lambda t}u(t)\} = \frac{n!}{(s-\lambda)^{n+1}}$

$\mathcal{L}\{tu(t)\} = \frac{1}{s^2}$ $\qquad \mathcal{L}\{x(t-t_0)\} = X(s)e^{-st_0}, t_0 \geq 0$

$\mathcal{L}\{t^n u(t)\} = \frac{1}{s^{n+1}}$ $\qquad \mathcal{L}\{sin(bt)u(t)\} = \frac{b}{s^2+b^2}$

$\mathcal{L}\{e^{\lambda t}u(t)\} = \frac{1}{s-\lambda}$ $\qquad \mathcal{L}\{cos(bt)u(t)\} = \frac{s}{s^2+b^2}$



Classes of complex exponential functions

Case 1:

$When\ s = 0, k \cdot e^{st} = k\ (a\ constant)$

Case 2:

$When\ \omega = 0, e^{st} = e^{\sigma t}\ (a\ monotonic\ exponential)$

Case 3: $When\ \sigma = 0, (sinusoidal)$

Case 4:No restriction, (exponentially varying sinusoid)

$Re[e^{st}] = e^{\sigma \cdot t}cos(\omega t)$

$Im[e^{st}] = e^{\sigma \cdot t}sin(\omega t)$

$x_e(t)$ is an even function of t if $x_e(t) = x_e(-t)$ ie $x_e(t)$ is symmetric about the vertical axis.

$x_o(t)$ is an odd function of t if $x_o(t) = -x_o(-t)$ ie $x_o(t)$ is anti-symmetric about the vertical axis.

Properties

1)even function x odd function = odd function

odd function x odd function = even function

even function x even function = even function

2)

a)$\int_{-a}^{a} x_e(t)dt = 2\int_0^a x_e(t)dt$

b)$\int_{-a}^{a} x_o(t)dt = 0$

Every signal $x(t)$ can be expressed as a sum of even and odd components:

$x(t) = x_e(t) + x_o(t)$

where

$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

Convolutions

$x_1 \circledast x_2 = \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau) \cdot d\tau$

$y_{ZS}(t) = x(t) \circledast h(t)$