$\frac{1}{j} = \frac{1}{\sqrt{-1}} \cdot \frac{\sqrt{-1}}{\sqrt{-1}} = \frac{\sqrt{-1}}{-1} = -j$

Polar Form

$c = r \cdot e^{j\theta} = r\angle\theta$

Conversion Between Rectangular and Polar

$r = \sqrt{x^2 + y^2}$ $y = r sin(\theta)$

$\theta = tan^{-1}(\frac{y}{x})$ $x = r cos(\theta)$

Euler's Formula:

$e^{j\theta} = cos(\theta) + j \cdot sin(\theta)$

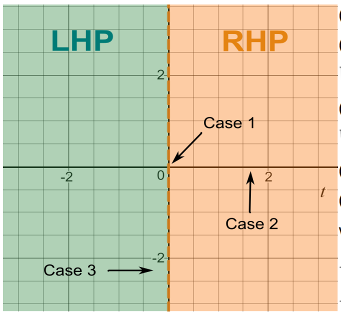
Useful Identities:

$e^{j90°} = e^{j\frac{\pi}{2}} = j$

$e^{j\pi} = -1$

For $c = x + jy = re^{j\theta}$

$c_* = complex\ conjugate\ of\ c = x - jy = re^{-j\theta}$



Classes of complex exponential functions

Case 1:

$When\ s = 0, k \cdot e^{st} = k\ (a\ constant)$

Case 2:

$When\ \omega = 0, e^{st} = e^{\sigma t}\ (a\ monotonic\ exponential)$

Case 3: $When\ \sigma = 0, (sinusoidal)$

Case 4: No restriction, (exponentially varying sinusoid)

$Re[e^{st}] = e^{\sigma \cdot t}cos(\omega t)$

$Im[e^{st}] = e^{\sigma \cdot t}sin(\omega t)$

For $s = \sigma + j\omega$

$s_* = complex\ conjugate\ of\ c = \sigma - j\omega$

$e^{s_* t} = e^{\sigma t}[cos(\omega t) - jsin(\omega t)]$

$e^{st} = e^{\sigma t}[cos(\omega t + jsin(\omega t)]$

$\frac{1}{2}[e^{st} + e^{s_* t}] = \frac{1}{2}[2e^{\sigma t}cos(\omega t)] = e^{\sigma t}cos(\omega t) = Re[e^{st}]$

$\frac{1}{2}[e^{st} - e^{s_* t}] = \frac{1}{2}[2e^{\sigma t}sin(\omega t)] = e^{\sigma t}sin(\omega t) = Im[e^{st}]$

$x_e(t)$ is an even function of t if $x_e(t) = x_e(-t)$ ie $x_e(t)$ is symmetric about the vertical axis.

$x_o(t)$ is an odd function of t if $x_o(t) = -x_o(-t)$ ie $x_o(t)$ is anti-symmetric about the vertical axis.

Properties

1) even function x odd function = odd function

odd function x odd function = even function

even function x even function = even function

2)

a) $\int_{-a}^{a} x_e(t)dt = 2\int_0^a x_e(t)dt$

b) $\int_{-a}^{a} x_o(t)dt = 0$

Every signal $x(t)$ can be expressed as a sum of even and odd components:

$x(t) = x_e(t) + x_o(t)$

where

$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

$\frac{f(x)}{x(x+a)(x+b)^2(x^2+c)} = \frac{A}{x} + \frac{B}{x+a} + \frac{C}{x+b} + \frac{D}{(x+b)^2} + \frac{Ex+F}{x^2+c}$

Delta Function Properties

a) $\delta(-t) = \delta(t)$

$x(t) \cdot \delta(t) = x(0)\delta(t)$

$x(t) \cdot \delta(t-T) = x(T)\delta(t-T)$

$\int_{-\infty}^{\infty} x(t)\delta(t-T)\,dt = x(T)$

b) $\int_{-\infty}^{\infty} x(t)\delta(t)\,dt = \int_{-\infty}^{\infty} x(0)\delta(t)\,dt$

$= x(0)\int_{-\infty}^{\infty} \delta(t)\,dt = x(0) \cdot 1$

$= x(0)$

Laplace transforms

$\mathcal{L}\{\delta(t)\} = 1$ $\mathcal{L}\{te^{\lambda t}u(t)\} = \frac{1}{(s-\lambda)^2}$

$\mathcal{L}\{u(t)\} = \frac{1}{s}$ $\mathcal{L}\{t^n e^{\lambda t}u(t)\} = \frac{n!}{(s-\lambda)^{n+1}}$

$\mathcal{L}\{tu(t)\} = \frac{1}{s^2}$ $\mathcal{L}\{x(t-t_0)\} = X(s)e^{-st_0}, t_0 \geq 0$

$\mathcal{L}\{t^n u(t)\} = \frac{1}{s^{n+1}}$ $\mathcal{L}\{sin(bt)u(t)\} = \frac{b}{s^2+b^2}$

$\mathcal{L}\{e^{\lambda t}u(t)\} = \frac{1}{s-\lambda}$ $\mathcal{L}\{cos(bt)u(t)\} = \frac{s}{s^2+b^2}$

$\mathcal{L}\{e^{-at}cos(bt)u(t)\} = \frac{s+a}{(s+a)^2+b^2}$

$\mathcal{L}\{e^{-at}sin(bt)u(t)\} = \frac{b}{(s+a)^2+b^2}$

$\mathcal{L}\{re^{-at}cos(bt+\theta)u(t)\} = \frac{r\ cos(\theta)s + (ar\ cos(\theta) - br\ sin(\theta))}{s^2+2as+(a^2+b^2)}$

$= \frac{0.5re^{j\theta}}{s+a-jb} + \frac{-0.5re^{j\theta}}{s+a+jb}$

$= \frac{As+B}{s^2+2as+c}$

Where

$r = \sqrt{\frac{A^2c+B^2-2ABa}{b^2}}$

$\theta = tan^{-1}(\frac{Aa-B}{Ab})$

$b = \sqrt{c = a^2}$

$\mathcal{L}\{e^{-at}[A\ cos(bt) + \frac{B-Aa}{b}sin(bt)]u(t)\} = \frac{As+B}{s^2+2as+c}$

Convolutions

$x_1 \circledast x_2 = \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau) \cdot d\tau$

$y_{ZS}(t) = x(t) \circledast h(t)$

$x(at), a \geq 0 \Leftrightarrow \frac{1}{a}X(\frac{s}{a})$

$x_1(t) \circledast x_2(t) \Leftrightarrow X_1(s)X_2(s)$

$x_1(t)x_2(t) \Leftrightarrow 12\pi j X_1(s) \circledast X_2(s)$

$x(0^+) \Leftrightarrow \lim_{s\to\infty} sX(s)(n > m)$

$x(\infty) \Leftrightarrow \lim_{s\to 0} sX(s)(poles of s(s) in LHP)$

Single-Input-Single-Output (SISO) System.

$x(t) \Rightarrow$[System]$\Rightarrow y(t)$

Linearity - A system is linear if it obeys superposition:

if $x_1 \Rightarrow y_1\ and\ x_2 \Rightarrow y_2$

then

$k_1x_1 + k_2x_2 \Rightarrow k_1y_1 + k_2y_2$

where k_1 and k_2 are constants

Zero Input Response: $x(t) = 0$

Zero State Response: $x_0 = 0$

Laplace Transform Properties:

$x_1(t) + x_2(t) \Leftrightarrow X_1(s) + X_2(s)$

$kx_1(t) \Leftrightarrow kX_1(s)$

$\frac{dx}{dt} \Leftrightarrow sX(s) - x(0^-)$

$\frac{d^2x}{dt^2} \Leftrightarrow sX(s) - x(0^-) - x'(0^-)$

$\frac{d^nx}{dt^n} \Leftrightarrow s^nX(s) - \sum_{k=1}^{n} s^{n-k}x^{k-1}(0^-)$

$\int_{0^-}^{t} x(\tau)d\tau \Leftrightarrow \frac{1}{s}X(s)$

$\int_{-\infty}^{t} x(t)d\tau \Leftrightarrow \frac{1}{s}X(s) + \frac{1}{s}\int_{-\infty}^{0^-} x(t)dt$

$x(t)e^{-s_0t} \Leftrightarrow X(s - s_0)$

$-tx(t) \Leftrightarrow dX(s)ds$

Forms of Euler's Formula:

$e^{jt} = cos\,t + j sin\,t$

$sin\,t = \frac{e^{jt} - e^{-jt}}{2j}$

$cos\,t = \frac{e^{jt} + e^{-jt}}{2}$

$sinc = \frac{sin(t)}{t}$

Exponential F.S.:

[Add definition of FS and exponential FS]

$\omega_n = \frac{2\pi}{T_0}(n+1)$

$x(t) = \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\omega_0 t}$

$D_n = \frac{1}{T_0}\int_{T_0} x(t) \cdot e^{-jn\omega_0 t}dt$

$D_0 = \frac{1}{T_0}\int_{T_0} x(t)dt = a_0 = c_0$

$D_{-n} = D_n^* = \frac{a_n + jb_n}{2}$

Amplitude Spectrum: $|D_n|\ \ vs.\ \ n \cdot \omega_0$

Phase Spectrum: $\measuredangle D_n\ \ vs.\ \ n \cdot \omega_0$

$|D_{-n}| = |D_n|$

$\measuredangle D_{-n} = -\measuredangle D_n$

Steady State:

$for\ x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cdot cos(n \cdot \omega_0 t + \theta_n)$

$y(t) = C_0|H(0)| + \sum_{n=1}^{\infty} C_n \cdot |H(jn\omega_0)|cos(n \cdot \omega_0 t + \theta_n + \measuredangle H(jn\omega_0))$

Amplitude Spectrum: $C_n \cdot |H(jn\omega_0)|\ \ vs\ \ n\omega_0$

Phase Spectrum: $\theta_n + \measuredangle H(jn\omega_0)\ \ vs\ \ n\omega_0$

Fourier Series:

$x(t) = a_0 + \sum_{n=1}^{\infty}[a_n cos(n\omega_0 t) + b_n sin(n\omega_0 t)]$

$\omega_n = \frac{2\pi}{T_0}(n+1)$

$a_0 = \frac{1}{T_0}\int_{T_0} x(t)dt$

$a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos(n\omega_0 t)\,dt$

$b_n = \frac{2}{T_0}\int_{T_0} x(t)\sin(n\omega_0 t)\,dt$

Compact Form:

$x(t) = c_0 + \sum_{n=1}^{\infty} c_n cos(n\omega_0 t + \theta_n)$

$c_0 = a_0, \omega_0 = \frac{2\pi}{T_0}$

$c_n\measuredangle\theta_n = a_n + jb_n, n \neq 0$

Given a BIBO system where the input is $x(t) = Acos(\omega_0 t + \phi)$ [As is the compact form of F.S.].

Note: improper fractions are ok for this situation.

$y(t) = A|H(j\omega_0)| \cdot cos(\omega_0 t + \phi + \measuredangle H(j\omega))$

$Where\ |H(j\omega_0)|$

$= [replace\ all\ multiplied\ values\ as\ complex\ numbers\ in\ H(j\omega_0)\ with\ distance\ formulas]$

$= \left\{(a + j\omega_0) \Rightarrow \sqrt{a^2 + \omega_0^2}\right\}$

and

$\measuredangle H(j\omega) = \tan^{-1}\left(\frac{\omega_0}{a}\right)$

Fourier Transforms

$X(\omega) = \mathcal{F}[x(t)] = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$

$x(t) = \mathcal{F}^{-1}[X(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$

Suffiscient Condition for FT Existance:

$\int_{-\infty}^{\infty} |x(t)|dt$

$x(t)$	$X(\omega)$	
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$a > 0$
$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$a > 0$
$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$	$a > 0$
$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$	$a > 0$
$t^ne^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$a > 0$
$\delta(t)$	1	
1	$2\pi\delta(\omega)$	
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
$\mathrm{sgn}\,t$	$\frac{2}{j\omega}$	
$\cos\omega_0 t\,u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	
$\sin\omega_0 t\,u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
$e^{-at}\sin\omega_0 t\,u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a > 0$
$e^{-at}\cos\omega_0 t\,u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a > 0$

$x(t)$	$X(\omega)$
$kx(t)$	$kX(\omega)$
$x_1(t)+x_2(t)$	$X_1(\omega)+X_2(\omega)$
$x^*(t)$	$X^*(-\omega)$
$X(t)$	$2\pi x(-\omega)$
$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
$x(t-t_0)$	$X(\omega)e^{-j\omega t_0}$
$x(t)e^{j\omega_0 t}$	$X(\omega-\omega_0)$
$x_1(t)*x_2(t)$	$X_1(\omega)X_2(\omega)$
$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$
$\frac{d^nx}{dt^n}$	$(j\omega)^nX(\omega)$
$\int_{-\infty}^{t} x(u)\,du$	$\frac{X(\omega)}{j\omega}+\pi X(0)\delta(\omega)$

$x(t)$	$X(\omega)$	
$\mathrm{rect}\left(\frac{t}{\tau}\right)$	$\tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$	
$\frac{W}{\pi}\mathrm{sinc}(Wt)$	$\mathrm{rect}\left(\frac{\omega}{2W}\right)$	
$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\mathrm{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
$\frac{W}{2\pi}\mathrm{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0 = \frac{2\pi}{T}$
$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Hyperbolic Trig Functions: Definitions

$sinh(t) = \frac{e^t - e^{-t}}{2}$

$csch(t) = \frac{1}{sinh(t)} = \frac{2}{e^t - e^{-t}}$

$cosh(t) = \frac{e^t + e^{-t}}{2}$

$sech(t) = \frac{1}{cosh(t)} = \frac{2}{e^t + e^{-t}}$

$tanh(t) = \frac{sinh(t)}{cosh(t)} = \frac{e^t - e^{-t}}{e^t + e^{-t}}$

$coth(t) = \frac{1}{tanh(t)} = \frac{e^t + e^{-t}}{e^t - e^{-t}}$

cosh2(t) - sinh2(t) = 1

tanh2(t) + sech2(t) = 1

coth2(t) - csch2(t) = 1

Inverse Hyperbolic Definitions

arcsinh(z) = ln(z + [sqrt](z2 + 1))

arccosh(z) = ln(z [sqrt](z2 - 1))

arctanh(z) = 1/2 ln((1+z)/(1-z))

arccsch(z) = ln((1+[sqrt](1+z2))/z)

arcsech(z) = ln((1[sqrt](1-z2))/z)

arccoth(z) = 1/2 ln((z+1)/(z-1))

Relations to Trigonometric Functions

sinh(z) = -j sin(iz)

csch(z) = i csc(iz)

cosh(z) = cos(iz)

sech(z) = sec(iz)

tanh(z) = -j tan(iz)

coth(z) = j cot(iz)

Amplitude spectrum: $|X(\omega)|\ vs\ n\omega_0$

$\mathcal{F} = [e^{-a|t|}] = \int_{-\infty}^{0} e^{at}e^{-j\omega t_0}dt + \int_0^{\infty} e^{-at}e^{-j\omega t_0}dt = \frac{2a}{a^2+\omega^2}$